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Evaluation of Various Techniques to Warm-Start a Successive Linear Programming Algorithm for Solving the IV ACOPF

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***Abstract*—Successive linear programming (SLP) has shown promising potential for solving the ACOPF problem. It is, however, essential to improve the computational performance of the algorithm. One way to achieve this goal is through initialization of the algorithm with a near-optimal solution. This paper examines various approaches to initialize the SLP-ACOPF, including uniform, flat, and warm start. AC- and DC-based techniques as well as a hybrid method are developed for warm-start initialization. The DC-based technique employs the solution to a DCOPF, while the AC-based methods employ the solution to a second order conic programming (SOCP) relaxation of the ACOPF. We also examine a hybrid method, where voltage magnitudes are obtained by SOCP and voltage angles are taken from DCOPF. The methods are tested on a range of systems, using off-the-shelf solvers, i.e., Gurobi and CPLEX, and the results are presented and compared.**

***Index Terms*—AC optimal power flow (ACOPF), successive linear programming (SLP), second order conic programming (SOCP), warm-start.**

I. Introduction

The objective of power system operation is to serve the customers at the minimum cost, taking into the consideration the physical constraints of the system. This can be expressed as an optimization problem, referred to as the optimal power flow (OPF). The OPF problem in its original form (ACOPF) is a nonlinear and non-convex optimization problem, which is challenging to solve [1], [2]. The challenges include the difficulty to identify a globally-optimal solution and more importantly its computational burden. Consequently, every single system operator in the U.S. chooses to employ one or another form a linearized version of the OPF problem, referred to as DCOPF [3], [4]. Since DCOPF abstracts from some of the complexities of the original problem, it does not produce a physically-feasible solution. Thus, the operators need to adjust the DCOPF solution, in order to achieve feasibility [5]. The simplifications embedded in the DCOPF along with such out of market adjustments lead to inefficiencies in the final solution. A report by the Federal Energy Regulatory Commission (FERC) estimates that the status quo may result in up to 10% additional cost, which can be avoided through efficient ACOPF solvers [2].

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The North American power grid is the most complex machine ever built by humans, with an annual economic sales value of more than $350 billion [6]. This implies that even a one percent improvement in the operation of the system can easily result in saving over 3 billion dollars a year. Since the result of DCOPF is likely not close to the globally-optimal solution to the original OPF, there is a growing interest among academic scholars to work around techniques that would make the ACOPF work fast enough for the real-time operations, and thereby operating the system more efficiently. The challenge remains to be the computational complexity of finding a quality solution within the limited available time.

Many methods have been proposed and tested for solving ACOPF, including a variety of convex relaxation techniques [7]–[10]. Another approach that is shown to perform well is the successive-linear programming (SLP) approximation of the currentvoltage (IV) ACOPF formulation [11]. This method takes advantage of the linear representation of network flows in a rectangular IV formulation, compared to a conventional quadratic power flow formulation in polar coordinate. SLP IV-ACOPF algorithm demonstrates promising scalability and performance properties [12].

The performance of SLP, in terms of convergence quality, objective value, and computational performance, largely depends on the initialization of the SLP algorithm. There are basically two types of initialization methods: 1) cold-start, which does not require a prior knowledge of the operating state; and 2) warm-start, which requires some knowledge of a quality solution, or a prior operating state.

This paper aims to improve the computational time required for solving the SLP IV-ACOPF problem. The contributions of this paper can be summarized as follows:

1) Three different approaches based on SOCP are developed and tested to warm-start the SLP method.
2) A hybrid SOCP-DCOPF method is developed and evaluated to initialize the SLP algorithm.
3) A few improvements are introduced to the SLP algorithm in order to reduce the run-time.

The rest of the paper is organized as follows. In Section II, we present and formulate an SLP algorithm for IV-ACOPF. This section also discusses uniform and flat starts. Different warm-start approaches are presented in Section III. Section

IV presents the computational performance of improved SLP algorithm with different initializations. A discussion of the results are provided in Section V, and finally conclusions are drawn in Section VI.

II. Successive Linear Programming

A. Algorithm Outline

Fig. 1 shows the flow-chart of the improved SLP IV-ACOPF.

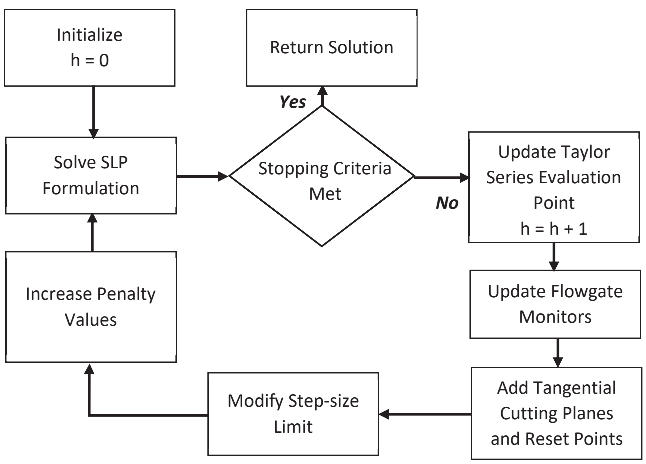

Fig. 1. The improved SLP IV-ACOPF algorithm.

The algorithm proposed by [12], originally, develops an SLP method for solving IV-ACOPF. Fig.1 is an extended process diagram of the algorithm described in [12] with two proposed improvements. Firstly, a step for increasing the penalty value is added in the algorithm, which incentivizes the iteration to quickly move toward a feasible solution. Secondly, the number of variables are significantly reduced, which ultimately improves the run-time by eliminating the unnecessary variables.

B. SLP Formulation

The SLP formulation presented in [12] is based on coupled model between voltage angles and magnitudes. The network model is used under the assumption of balanced three-phase network operating under steady-state conditions. The nonlinear IV-ACOPF and the subsequent LP sub-problem is formulated in rectangular coordinates for the voltage phasor $v_n = v_n^r + jv_n^j$ at each bus $n \in \mathcal{N}$, the current phasor $i_n = i_n^r + ji_n^j$ at each bus $n \in \mathcal{N}$, and the current $i_{k(.)} = i_{k(.)}^r + ji_{k(.)}^j$ on all network flows $k(.) \in \mathcal{F}$.

In order to linearize the quadratic cost function of the generators in objective function of optimal power flow problem, a pice-wise linear interpolation is applied, and this approach yields a tighter upper bound on the quadratic cost function since typically generators has a monotonically increasing offer curves [12]. Hence, the objective function becomes as shown in (1)

$$\begin{aligned}\min \sum_{g\in\mathcal{G}}\sum_{l\in\mathcal{L}} & \left[\left(C_{n,l}^g p_{n,l}^g \right) + C_g^0 \right] \\ & + \sum_{n\in\mathcal{N}} \Big[P^\epsilon \left(p_n^{viol,-} + p_n^{viol,+} \right) + Q^\epsilon \left(q_n^{viol,-} + q_n^{viol,+} \right) \\ & + V^\epsilon \left(v_n^{viol,-} + v_n^{viol,+} \right) \Big] \\ & + \sum_{k\in\mathcal{K}} \left[I^\epsilon \left(i_{k(n,m)}^{viol,+} + i_{k(m,n)}^{viol,+} \right) \right] \end{aligned} \tag{1}$$

The nonlinear terms in some constraints of a nonlinear IV-ACOPF formulation in rectangular coordinates are addressed by applying first order Taylor series approximation. To approximate the first order linearization, Taylor series evaluation points, which are denoted with a "hat" are used for iteration h [12].
subject to:

$$p_n^g = \sum_{g\in\mathcal{G}} p_{n,l}^g + P_n^{min} \tag{2}$$

$$0 \le p_{n,l}^g \le P_n^g \tag{3}$$

$$P_n^{min} \le p_n^g \le P_n^{max} \tag{4}$$

$$Q_n^{min} \le q_n^g \le Q_n^{max} \tag{5}$$

$$v_n^{sq} = 2\hat{v}_n^{r(h)} v_n^r + 2\hat{v}_n^{j(h)} v_n^j - (\hat{v}_n^{r(h)})^2 - (\hat{v}_n^{j(h)})^2 \tag{6}$$

$$\begin{aligned} p_n^g &= \hat{v}_n^{r(h)} i_n^r + \hat{v}_n^{j(h)} i_n^j + v_n^r \hat{i}_n^{r(h)} + v_n^j \hat{i}_n^{j(h)} \\ &\quad - \hat{v}_n^{r(h)} \hat{i}_n^{r(h)} - \hat{v}^{j(h)} \hat{i}_n^{j(h)} + P_n^d \end{aligned} \tag{7}$$

$$\begin{aligned} q_n^g &= \hat{v}_n^{j(h)} i_n^r - \hat{v}_n^{r(h)} i_n^j + v_n^j \hat{i}_n^{r(h)} - v_n^r \hat{i}_n^{j(h)} \\ &\quad - \hat{v}_n^{j(h)} \hat{i}_n^{r(h)} + \hat{v}^{r(h)} \hat{i}_n^{j(h)} + P_n^d \end{aligned} \tag{8}$$

$$i_n^r = \sum_{k(n,\cdot)} i_{k(n,m)}^r + G_n^{sh} v_n^r - B_n^{sh} v_n^j \tag{9}$$

$$i_n^j = \sum_{k(n,\cdot)} i_{k(n,m)}^j + G_n^{sh} v_n^j + B_n^{sh} v_n^r \tag{10}$$

$$P_n^{min} - p_n^{viol,-} \le p_n^g \tag{11}$$

$$p_n^g \le P_n^{min} + p_n^{viol,+} \tag{12}$$

$$Q_n^{min} - q_n^{viol,-} \le q_n^g \tag{13}$$

$$q_n^g \le Q_n^{min} + q_n^{viol,+} \tag{14}$$

$$(V_n^{min})^2 - v_n^{viol,-} \le v_n^{sq} \tag{15}$$

$$v_n^{sq} \le (V_n^{min})^2 + v_n^{viol,+} \tag{16}$$

$$\hat{v}_n^{r(h)} - V_n^{(h)} \le v_n^r \le \hat{v}_n^{r(h)} + V_n^{(h)} \tag{17}$$

$$-V_n^{max} \le v_n^r \le V_n^{max} \tag{18}$$

$$\hat{v}_n^{j(h)} - V_n^{(h)} \le v_n^j \le \hat{v}_n^{j(h)} + V_n^{(h)} \tag{19}$$

$$-V_n^{max} \le v_n^j \le V_n^{max} \tag{20}$$

$$0 \le p_n^{viol,-} \tag{21}$$

$$0 \le p_n^{viol,+} \tag{22}$$

$$0 \le q_n^{viol,-} \tag{23}$$

$$0 \le q_n^{viol,+} \tag{24}$$

$$0 \le v_n^{viol,-} \tag{25}$$
$$0 \le v_n^{viol,+} \tag{26}$$
$$i^r_{k(n,m)} = \mathcal{R}\bigg(|\tau_{kn}|^2(Y_k + Y_{kn}^{sh})\mathbf{v}_n - \tau_{kn}^* Y_k \mathbf{v}_m\bigg) \tag{27}$$
$$i^j_{k(n,m)} = \mathcal{I}\bigg(|\tau_{kn}|^2(Y_k + Y_{kn}^{sh})\mathbf{v}_n - \tau_{kn}^* Y_k \mathbf{v}_m\bigg) \tag{28}$$
$$i^r_{k(m,n)} = \mathcal{R}\bigg(-\tau_{kn} Y_k \mathbf{v}_n + (Y_k + Y_{km}^{sh})\mathbf{v}_m\bigg) \tag{29}$$
$$i^j_{k(m,n)} = \mathcal{I}\bigg(-\tau_{kn} Y_k \mathbf{v}_n + (Y_k + Y_{km}^{sh})\mathbf{v}_m\bigg) \tag{30}$$
$$i^{sq}_{k(\cdot)} = 2\hat{i}^{r(h)}_{k(\cdot)} i^r_{k(\cdot)} + 2\hat{i}^{j(h)}_{k(\cdot)} i^j_{k(\cdot)} - (\hat{i}^{r(h)}_{k(\cdot)})^2 - (\hat{i}^{j(h)}_{k(\cdot)})^2 \tag{31}$$
$$i^{sq}_{k(\cdot)} \le (I_k^{max})^2 + i^{viol,+}_{k(\cdot)} \tag{32}$$
$$-I_k^{max} \le i^r_{k(\cdot)} \le I_k^{max} \tag{33}$$
$$-I_k^{max} \le i^j_{k(\cdot)} \le I_k^{max} \tag{34}$$
$$0 \le i^{viol,+}_{k(\cdot)} \tag{35}$$
$$\hat{v}^{r(h)} v^r + \hat{v}^{j(h)} v^j \le (V^{max})^2 + v^{viol,+} \tag{36}$$
$$\hat{i}^{r(h)} i^r + \hat{i}^{j(h)} i^j \le (I^{max})^2 + i^{viol,+} \tag{37}$$

C. Uniform Start

The uniform starts assume that $v_n^r \backsim \mathcal{U}(V_n^{min}, V_n^{max})$ and $v_n^j = 0$ for all $n \in \mathcal{N}$; given that the uniform start does not incorporate any knowledge of a prior operating state, it is by definition a cold start.

D. Flat Start

The flat start assumes voltages to be equal to 1 p.u. at all nodes, while assuming zero phase angle for all buses. Given that the flat start does not incorporate any knowledge of a prior operating state, it is by definition a cold start.

III. Warm Start

In warm-start, we run a relaxed ACOPF, and the results (i.e. either nodal voltage magnitudes, nodal voltage angle, or both) are used in the initialization step for the SLP approach. The warm-start can be examined in three categories:

1) DCOPF-based start;
2) SOCP-based start;
   a) Both nodal voltage magnitudes and angles are obtained from SOCP;
   b) Nodal Voltage magnitudes are obtained from SOCP and their phase angles are assumed to be zero;
   c) Nodal Voltage magnitudes are obtained from SOCP and their phase angles are obtained from the results of running a power flow;
3) Hybrid DCOPF-SCOP Start.

A. DCOPF Warm Start

In this approach for SLP initialization, first, the DCOPF is solved, using (38) as objective function.

$$\min \qquad \sum_{g \in \mathcal{G}} \sum_{l \in \mathcal{L}} \bigg[\Big(C_{n,l}^g p_{n,l}^g\Big) + C_g^0\bigg] \tag{38}$$

subject to (2), (3), (4), and

$$-S_k^{max} \le P_{ij} \le -S_k^{max} \tag{39}$$
$$p_j - p_j^d = \sum_{k:j \to k} P_{jk} - \sum_{i:i \to j} P_{ij} \tag{40}$$
$$P_{ij} = b_{ij}(\theta_i - \theta_j) \tag{41}$$

The solution θ_i^* is fed to the SLP initialization as $\angle V_i$, and nodal voltage magnitudes $|V_i|$ are assumed to be 1 p.u. for all buses.

B. SOCP Warm Start

In ACOPF SOCP relaxation, we use branch flow model with SOCP relaxation [9], [13]. Similar to DCOPF we first solve the SOCP relaxation of ACOPF, for which we use the same objective function as in DCOPF (i.e. (38))
subject to (2), (3), (4), and

$$p_j - p_j^d = \sum_{k:j \to k} P_{jk} - \sum_{i:i \to j} (P_{ij} - r_{ij} l_{ij}) - g_j v_j, \ \forall j \tag{42}$$
$$q_j - q_j^d = \sum_{k:j \to k} Q_{jk} - \sum_{i:i \to j} (Q_{ij} - x_{ij} l_{ij}) - b_j v_j, \ \forall j \tag{43}$$
$$v_j = v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij}, \ \forall (i,j) \in E \tag{44}$$
$$v_i l_{ij} \ge P_{ij}^2 + Q_{ij}^2, \ \forall (i,j) \in E \tag{45}$$

where P_{ij} =Re$[S_{ij}]$, Q_{ij} =Im$[S_{ij}]$, $l_{ij} = |I_{ij}|^2$, $v_i = |V_i|^2$, p_i =Re$[s_i]$, and q_i =Im$[s_i]$.
The nodal voltages can be obtained by

$$|V_i| = \sqrt{v_i^*} \tag{46}$$

Whereas, the nodal voltage angles can be calculated as

$$\theta_{n \times 1} = A^+_{k \times n} \Delta\theta_{k \times 1} \tag{47}$$

$A^+_{k \times n}$ is the pseudo inverse matrix of adjacency matrix $A_{k \times n}$, and $\theta_1 = 0$.
$\Delta\theta_{k \times 1}$ can be calculated using the equation (48)

$$\Delta\theta_{k \times 1} = \frac{|V_i|^2 + |V_j|^2 - |I_{ij}||Z_{ij}|}{2|V_i||V_j|} \tag{48}$$

1) $SOCP_1$ Start: In this approach, the $|V_i|$ and $\angle V_i$ needed for initializing the SLP algorithm is both obtained from SOCP relaxation formulation, which is the minimization of (38) subject to (2), (3), (4), (42) - (45). After solving SOCP relaxation, the nodal voltage magnitudes can be obtained using (46) and their corresponding angles can be obtained using (47). The disadvantage of this initialization method is that it takes visible time for large cases to calculate the $A^+_{k \times n}$, which is needed for recovering $\angle V_i$ for each bus.

2) $SOCP_2$ Start: In this approach, $|V_i|$ needed for initializing the SLP method is obtained from SOCP relaxation formulation, which is the minimization of (38) subject to (2), (3), (4), (42) - (45). After solving SOCP relaxation problem, the nodal voltage magnitudes can be obtained using (46). However, unlike $SOCP_1$ start, the $\angle V_i$ in this approach is

TABLE I

SLP OBJECTIVE FUNCTION VALUES FOR DIFFERENT BENCHMARK CASE STUDIES USING DIFFERENT INITIALIZATION TECHNIQUES.

Cases	Flat	Uniform	DCOPF	$SOCP_1$	$SOCP_2$	$SOCP_3$	SOCP+ DCOPF
IEEE 14 bus	$8,091.38	$8,101.74	$8,090.97	$8,090.92	$8,090.86	$8,151.52	$8,095.93
IEEE 30 bus	$577.47	$577.49	$577.37	$577.75	$577.47	$577.49	
IEEE 57 bus	$41,763.36	$41,770.89	$41,762.68	$41,779.15	$41,777.50	$41,778.69	$41,798.02
IEEE 118 bus	$130,064.87	$130,045.31	$130,452.85	$130,112.08	$130,162.10	$129,930.05	$130,431.07
IEEE 300 bus	$720,422.26	$721,536.05	$721,229.40	$723,846.30	$720,918.99	$720,122.61	$729,869.52
case3120sp	$2,141,843.20	$2,142,927.77	$2,147,241.49	**	$2,142,623.65	$2,142,186.80	$2,150,969.10
case6515rte	$109,566.84*	#	$110,181.18*	#	#	$109,579.05	#

* Max 50 iterations reached ** Quieting without a feasible solution # Couldn't solve within one hour

TABLE II

SLP RUNTIME IN SECONDS FOR DIFFERENT BENCHMARK CASE STUDIES USING DIFFERENT INITIALIZATION TECHNIQUES.

Cases	Flat	Uniform	DCOPF	$SOCP_1$	$SOCP_2$	$SOCP_3$	SOCP+ DCOPF
IEEE 14 bus	0.14	0.15	0.14	0.12	0.12	0.10	0.05
IEEE 30 bus	0.18	0.19	0.20	0.182	0.17	0.16	
IEEE 57 bus	0.25	0.24	0.24	0.22	0.22	0.20	0.16
IEEE 118 bus	0.63	0.62	0.66	0.61	0.59	0.55	0.54
IEEE 300 bus	2.31	2.51	3.77	2.48	1.97	2.03	2.47
case3120sp	265.00	320.00	188.00	**	239	182	171
case6515rte	2330.00	#	1970.00	#	#	1680	#

** Quieting without a feasible solution # Couldn't solve within one hour

assumed to be zero. This approach will save the run-time for calculating the $A^+_{k\times n}$ and recovering angles, which could be significant for large bus case systems.

3) $SOCP_3$ Start: In this approach, the $|V_i|$ and $\angle V_i$ needed for initializing the SLP solution is both obtained from a fast decoupled power flow using the formulation in [14]. p^g_n and q^g_n input for the fast decoupled power flow is obtained from SCOP relaxation formulation for solving ACOPF, which is the minimization of (38) subject to (2), (3), (4), (42) - (45).

C. Hybrid SOCP-DCOPF Warm-Start

In this approach, the $|V_i|$ needed for initializing the SLP solution is obtained from SOCP relaxation formulation, which is the minimization of (38) subject to (2), (3), (4), (42) - (45). After solving SOCP relaxation optimization, the nodal voltage magnitudes can be obtained using (46). $\angle V_i$ can be obtained using DCOPF, where the same (38) is solved as objective function subject to (2), (3), (4), (39) (40), (41). For the sake of improving the run-time of overall solution, SOCP and DCOPF can be run in parallel as the two are independent of each other.

IV. Case Studies

We used Java to implement all formulations, and employed eclipse to solve each instance. In this computational study, eclipse employs CPLEX 12.71 and Gurobi 7.51 as second order conic programming and linear programming solvers. All computational experiments have been carried out on a HP ENVY Desktop 750-524 with Intel(R) Core(TM) i7-7700 3.60 GHz 8-Core Processors, 16GB RAM, and the Windows 10 operating system using a single thread.

Different test systems such as IEEE benchmarks, a polish system and RTE systems have tested for various SLP initialization approaches. As for the IEEE benchmarks, the 14- bus, 30-bus, 57 bus, 118-bus, and 300-bus test cases were solved. In polish system, 3120 bus system and in RTE case6515rte have been tested for various SLP initialization approaches.

V. Results

This section presents a comparison of performance between different SLP initialization techniques for different test cases. Table I compares the objective function values for different benchmark case studies using different initialization techniques. This table provides an insight on how different start approach can affect the objective function value. Table II compares the SLP runtime for different benchmark case studies using different initialization techniques. This table provides an insight on how different start approach can affect the runtime and subsequently the convergence quality of SLP formulation. Fig. 2 picks the IEEE 118-bus test system and demonstrates the impact of different SLP initialization techniques on convergence quality, objective value, and computational performance.

VI. Discussion

By looking at the results, particularly Fig. 2 the impact of different initialization techniques for SLP on its convergence quality, objective value, and computational performance can be clearly observed. There are two factors to the initialization technique that affects the performance of the SLP algorithm: the nodal voltage magnitudes $|V_i|$ and nodal voltage angles $\angle V_i$. The larger the system, the more significant will be the effects, particularly on the runtime. SOCP relaxation can provide a feasible voltage magnitude set $|V_i|$; however, the nodal voltage angles $\angle V_i$ provided by SOCP relaxation are far away from the angle values in the feasible solutions. Fig. 1

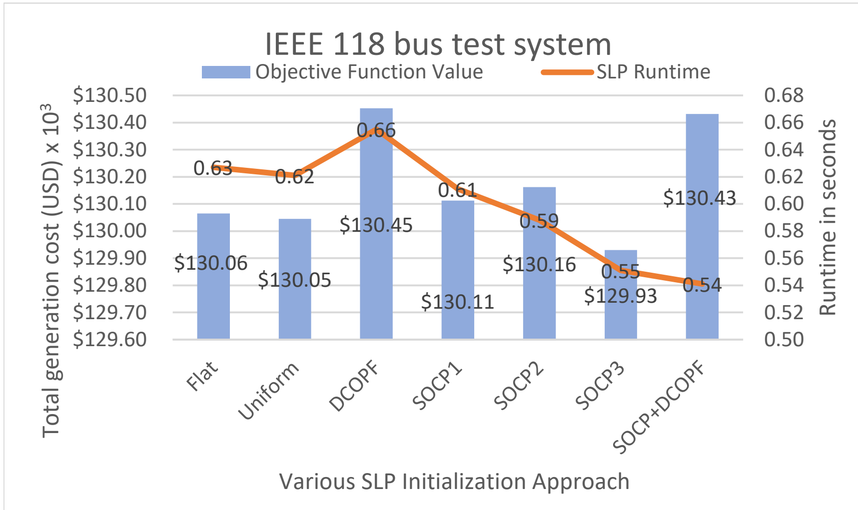

Fig. 2. The impact of different SLP initialization techniques on the performance of SLP algorithm for IEEE 118-bus test system.

shows that the objective function value and the runtime depend on the voltage input. As the initial solution gets closer to the final solution (both in terms of feasibility and optimality), the final value of the objective function improves and the runtime becomes smaller. In this context, $SOCP_3$ which takes the voltage magnitudes from SOCP and voltage angles after running a full AC power flow (i.e. an input close to feasible solution) provides the best objective function value and runtime.

TABLE III
RUNTIME IN SECONDS FOR OBTAINING $|V_i|$ AND $\angle V_i$ FROM SOCP INCLUDING THE SOCP TIME

Cases	$SOCP_1$	$SOCP_2$	$SOCP_3$	SOCP+ DCOPF
IEEE 14 bus	0.0526	0.0501	0.236	0.127
IEEE 30 bus	0.0505	0.0505	0.148	0.0505
IEEE 57 bus	0.0631	0.0633	0.272	0.143
IEEE 118 bus	0.148	0.147	0.429	0.236
IEEE 300 bus	0.514	0.507	1.30	0.597
case3120sp	2.13	2.13	3.66	2.13
case6515rte	16	16	16.8	16

VII. CONCLUSION

This paper examines various approaches to start a successive linear programming (SLP) algorithm for solving alternating current optimal power flow (ACOPF). Moreover, a number of improvements are proposed for the original SLP IV-ACOPF method. The initialization approaches have been tested on a range of test systems with different sizes ranging from 14-bus to 6515 buses. The results demonstrate the $SOCP_3$ yields the the fastest run time and the best objective function value for most of the test cases. This method takes the voltage magnitudes from SOCP and voltage angles after running an AC power flow. The performance superiority becomes more apparent as the size of the systems increase, which emphasizes the potential for real-world large-scale systems.

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