Tractable Stochastic Unit Commitment for Large Systems during Predictable Hazards

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***Abstract*—Many predictable natural hazards, including extreme weather events, lead to outage of multiple transmission lines. Although such outages can be predicted in advance, there is a great deal of uncertainty in these predictions. To appropriately use the failure estimations in day-ahead power system scheduling, this paper formulates a stochastic unit commitment (SUC) problem. The formulated problem, however, is extremely computationally-demanding, as the uncertainty is placed on the binary status of transmission lines. This paper, then, develops a computationally efficient algorithm to solve the formulated SUC for large-scale systems. The algorithm employs generation shift factors to enable rapid calculation of power flows. Additionally, flow canceling transactions are used to model multiple line outages without having to recalculate shift factors. Finally, critical constraints are iteratively detected and added to the problem. This approach substantially reduces the size of the problem, which helps computational tractability. The effectiveness of the developed algorithm is demonstrated through a simulation study on Texas 2000-bus test system. The algorithm is used to minimize the lost load during a hypothetical hurricane that passes through the system. In comparison with the $b-\theta$ formulation, the proposed algorithm can solve the problem up to 98% faster and with 95% less memory.**

***Index Terms*—Extreme weather, predictable natural hazards, preventive operation, power system reliability, power system resilience, stochastic unit commitment, uncertainty management.**

I. Nomenclature

A. Sets

G	Set of generators
g	Index of the generator, $g \in G$
N	Set of buses
n	Index for the bus, $n \in N$
K	Set of transmission lines and transformers
k	Index of the transmission line and transformers, $k \in K$
M	Set of monitored transmission lines and transformers
m	Index of the monitored transmission line, $m \in M$
S	Set of Scenarios
s	Index of the scenario, $s \in S$
O	Set of outages
o	Index of outage, $o \in O$
frm	Set of starting bus of lines
to	Set of ending bus of lines

B. Parameters

ng	Number of generation units
nl	Number of transmission lines and transformers
nb	Number of buses
ns	Number of scenarios
c	Cost of generation
c^{NL}	No-load cost for generator
c^{SU}	Start-up cost for generator
c^{SD}	Shut-down cost for generator
c^{lsh}	Load shedding cost (penalty)
c^{og}	Over-generation cost (penalty)
π	Scenario possibility
PG^{max}	Maximum generation power by generator
PG^{min}	Minimum generation power by generator
F^{max}	Maximum thermal capacity of line
F^{min}	Minimum thermal capacity of line
Rmp^{HR}	Hourly ramp-up and ramp-down of generator

C. Variables

PG	Generated power by gnerator
P^{d}	Demand power at bus
P^{lsh}	Load shedding power
P^{og}	Over-generation power
$\boldsymbol{PTDF}$	Power transfer distribution factor matrix
$\boldsymbol{LODF}$	Line outage distribution factor matrix
$\boldsymbol{P}$	Net nodal injected power matrix
$\boldsymbol{F}$	Line flow vector
$\boldsymbol{A}$	Adjacency matrix
$\boldsymbol{B}_{br}$	Branch admittance matrix
$\boldsymbol{B}$	Nodal admittance matrix
$\mathbf{b}$	Susceptance of line
$\boldsymbol{FC}$	Flow canceling transactions vector
u	Unit commitment binary variable
v	Start-up binary variable
x	Shut-down binary variable

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II. INTRODUCTION

UNIT commitment (UC) is frequently used by power system operators to identify the commitment status and dispatch of the generating units in order to serve the load at the minimum cost [1]. System operators employ UC for day-ahead and intra-day operations as well as forward market clearing. UC also performs reliability and validity assessments [2]. Despite all the enhancements in UC solvers over the last decades, it is still considered to be a computationally demanding problem, due to the inclusion of integer variables describing the commitment status of the generating units [3]–[5]. The computational burden increases with the size of the system as well as the modeling of uncertainties. Recently, to enhance the system reliability in the presence of high penetration of renewable resources, additional sources of uncertainty and risk are proposed to be included in the UC problem. Such model enhancements, while improving the system efficiency, will make the UC problem even more challenging to solve [2].

While a vast body of the literature has focused on uncertainties associated with renewable energy [6]–[10], predictable hazards are another category of uncertainties, which are not yet properly integrated into UC. Severe weather, such as hurricanes, tornadoes, ice storms, and snowstorms, are examples of such hazards, which can be predicted in advance. Over the last few years, the damage to the transmission and distribution networks, caused by hurricanes and tornadoes, has been responsible for over 80% of power outages in the United States, affecting many millions of people [11], [12]. Severe weather usually leads to outage of multiple transmission and distribution elements, whose failure probabilities can be predicted. However, as the number of the affected elements is large and the outages change the network topology over time, inclusion of such probabilities within the UC problem will make it extremely hard to solve using existing formulations.

Traditionally, deterministic methods based on the operating reserve requirements have been used to solve the unit commitment problem in the presence of uncertainties. In deterministic methods, a minimum level of reserve is obtained for uncertainty management [13]. Although the deterministic methods are computationally efficient, they do not necessarily guarantee reliability and can be economically inefficient. Moreover, the current industry practices focus on *N*-1 reliability and are neither designed nor capable of handling multiple line outages. It should be noted that with the current deterministic reserve requirements, reserve deliverability is not guaranteed even for an *N*-1 event [14]. Hence, there is a need for efficiently modeling the uncertainties and risks of multiple line outages, during a predictable hazard, such as a hurricane.

An alternative approach to deterministic methods is stochastic optimization. A scenario-based stochastic optimization problem models future uncertainties as possible scenarios, with a realization probability. As stochastic optimization explicitly models uncertainties, it can achieve higher levels of efficiency; this gain in solution quality, however, comes at the cost of substantial computational burden [1], [2]. This paper precisely addresses this challenge by developing a computationally efficient algorithm for stochastic unit commitment (SUC), considering uncertain *N-k* line outage scenarios. The method developed in this paper, thus, can be used to solve preventive unit commitment, in an attempt to reduce power outages.

Since the introduction of SUC, many researchers have developed a variety of methods to model more uncertainties and improve SUC's computational efficiency [15]–[19]. The computational demands of the problem can vary based on the type of uncertainty, e.g., generation dispatch or transmission status, and the uncertainty distribution, e.g., normal or Boolean. As mentioned earlier, this paper focuses on the uncertainty associated with the status of transmission elements.

There are a few studies on the modeling of uncertain equipment failures in the transmission network. A method to solve the security-constrained UC for large networks with one line outage possibility is developed in [20]. However, the proposed method is not valid when there are multiple line outages. Another relevant work in terms of the formulation is the transmission topology optimization problem [21], where a number of transmission lines are switched out. The formulation in [21] is of particular interest, due to the derivation of flow canceling transactions. The formulation allows computation of network flows, in the presence of multiple line outages without requiring recalculation of the shift factor matrix. We adopt this technique in our paper to improve the computational efficiency of the model. Another important observation in the literature is presented in [22], where the authors suggest that combining stochastic methods with deterministic methods is advantageous as the deterministic method can compensate the unseen uncertainties. Although this approach will enhance the solution time, the problem can still be too hard to solve even with a few scenarios. This is shown later in the results section of this paper.

This paper contributes to the literature by developing a computationally efficient stochastic unit commitment formulation, which can handle uncertain *N*-k line outage scenarios on a large-scale system. The developed model can be adopted for preventive operation during severe weather such as hurricanes or ice-storms or any other predictable natural/man-made hazards that can damage transmission lines. The model is capable of handling large-scale systems with standard hardware within an acceptable time.

The remainder of this paper is organized as follows: in Section III, our basic targets and summery of contributions are introduced. Section IV represents our methodology, which is a discussion on selected methods and models that are used in this study. In Section V, our algorithm and overall flowchart for calculations are described, and the mathematical formulation of the problem is explained. Our test-case study, used to evaluate the performance and accuracy of the proposed model, is discussed in Section VI. Finally, Section VI,VII, and VIII present the conclusions of this study and opportunities for future research.

III. ASSUMPTIONS AND CONTRIBUTIONS

In this paper, for the first time, we develop a formulation that enables SUC to address multiple line outages in the network.

The formulation can be used in different applications such as SUC in the presence of hurricane, reliability analysis, and line switching optimization studies. The model uses a smart iterative technique in combination with SUC in order to eliminate unnecessary variables and constraints to make the calculation faster with minimum hardware. It should be noted that in this paper, "scenario" refers to any particular future condition that can happen due to the presence of uncertainties in the network. The targets that we aim to achieve through the developed model are summarized below:

1- The method should be able to solve the SUC for a large-scale real-world size network with multiple scenarios.
2- The method should be able to solve within an acceptable computational time with standard hardware.
3- As multiple line outages are allowed, the network topology can change over the duration of the study (a day), and the model should be able to handle this.
4- As changes in the topology may lead to inevitable load shedding or over-generation (outage of a radial line), the model should allow relaxation of nodal power balance.

IV. Methodology

In this section, in order to make the formulation easier to understand and follow, the process of method evaluation and selection is described step by step. First, we need to evaluate existing methods for each sub-problem and select an appropriate one or develop a new method, if there is no existing solution. Second, all sub-problems should be combined to form the complete SUC model. Later, the algorithm should be fine-tuned for enhanced computational time and hardware requirements.

A. Main Method Selection

The main goal here is to optimize the objective function, which is defined as the generation cost or load shedding and over-generation penalty, in the presence of uncertain multiple line failure scenarios during the day. As discussed earlier, for this application, deterministic methods are not efficient in term of economic efficiency and reliability. Robust optimization, as introduced and used in [23]–[25], is not efficient either due to two main shortcomings. First, in robust unit commitment, usually the worst possible case is considered. As there are many components (lines) with outage possibility, deciding based on the worst possible case would impose unnecessary cost through the tighter constraints to the system. Second, it is not easy to determine the worst case when there are multiple outages. In other words, it is not necessarily a correct assumption that any line with any chance of failure should be considered faulty to create the worst possible case. For example, if there are two parallel lines with failure chances, losing both may be better than losing only the stronger one, in terms of congestion and transfer capability. Moreover, while robust optimization may reduce load shedding, it can increase the over-generation. The two remaining candidate methods are stochastic unit commitment and dynamic optimization. Considering all the advantages and disadvantages of these two methods as described in [2], here, we choose stochastic optimization as our modeling approach.

B. Power Flow Modeling.

Traditionally, there are two main methods to calculate the power flows in the network: the full AC model and DC approximations. At the moment, most AC optimal power flow solvers cannot guarantee the optimality of the solution and take very long to solve even for small systems [26], [27]. Consequently, every single system operator in North America uses one or another form of DC optimal power flow in their operation software [28]. Thus, DC power flow approximation is used in this paper. While the DC-based, B-θ formulation is well-known in academic studies as described in [28], [29], we decided not to use it due to its computational inefficiency. The B-θ formulation calculates the voltage angles for all the buses and all the hours individually to calculate the line flows. This translates to a large number of unnecessary variables and constraints in the model that leads to increased computational burden.

The *injection shift factor* and *power transfer distribution factor* (PTDF) are known as promising methods, which avoid unnecessary calculations [21], [30]. Injection shift factor determines the flow of any line when the net injection at all buses is known. PTDF represents the sensitivity of flow of any line with respect to a transfer of power between two buses. The net injection power at each bus can be assumed as a transfer from that bus to the slack bus. Therefore, PTDF can be used to determine the line flow when the slack bus is excluded and injected power at each bus is known. Note that PTDF matrix is a network describer, independent from operation point. PTDF needs to be calculated once for a network and not per operation point. Here, we use a modified version of the PTDF method to calculate power flows, as described in section V.

C. Multiple Line Outage Modeling

In the standard PTDF method, a change in the network topology requires recalculation of the PTDF matrix. To have a picture of how much time PTDF calculation requires, in our results for large-scale networks, PTDF calculation consumes around 40% of total solution time when the model solves the standard unit commitment problem, with no uncertainty modeling, for 24 periods. Thus, in SUC, as the network topology can change any time, it is not efficient to recalculate the PTDF. A single line outage can be handled simply using the famous *line outage distribution factors* (LODF) [31]. Numerous studies use this technique to calculate the power flow with different applications with a single line outage [32], [33]. However, the standard formulation is not valid in the case of multiple line outages.

LODF identifies the percentage of the flow on a line that would flow on other lines, should the first line go out of the service. Similar to PTDF, LODF also depends on the network topology, and one cannot apply superposition to calculate the flows after the outage of two or more lines.

In the formulation section, using the original idea of LODF and flow canceling transactions concept, developed in [21], we develop a formulation to model power flows with multiple line outages, without requiring recalculation of the PTDF matrix.

V. Algorithm and Formulation

A. Algorithm

For large networks, the mathematical representation of SUC will include too many variables and constraints. A properly designed formulation should reduce the required calculation by avoiding a large number of unnecessary variables and constraints without changing the underlying problem. This paper aims to offer a highly efficient formulation that avoids unnecessary calculations using an iterative approach by only including the necessary variables and constraints.

To do so, the main calculations of SUC are divided into three segments, as shown in Fig. 1. In the first segment, block "A" reads the data from stored files, and block "B" calculates network parameters that will not be changed over the course of the study. The stored files include information on buses, lines, loads, generators, outages, and failure scenarios. The two main network parameters that will not change over time include **PTDF** and **LODF** matrices. At the end of this segment, the parameters that define the constraints are set for the first iteration of SUC to exclude all the line thermal capacity limits and generation ramp constraints.

At the beginning of the second segment, block "C" reads control flags and generates constraints for the current iteration of the SUC, based on these flags. Note that the flags are all initialized to 0 for thermal and ramping constraints, at the end of segment 1. These flags indicate the constraint/variable that should or should not be added. For example, $M_{(s)}$ is a control matrix which includes all the lines that should be monitored under scenario s, and $O_{(s)}$ reflects the failed lines under scenario s.

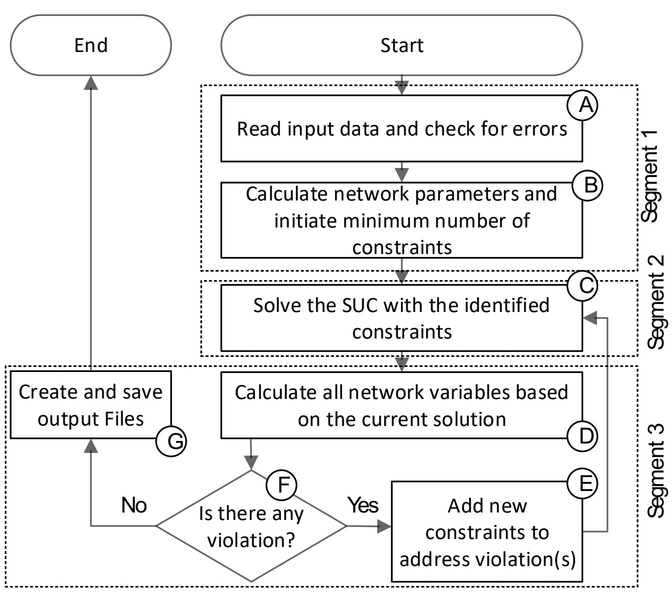

Fig. 1. Simplified flowchart of calculations

Finally, in the third segment, with the calculated solution from segment two, all network variables are calculated in block "D," and block "E" and "F" check if the values are out of bound and any more constraints are needed to be added to the SUC in the next iteration. If the optimal solution is found and no constraint is violated, block "G" creates the output results.

This approach essentially removes a large number of unnecessary variables and constraints from the optimization model. The variables are calculated, and the constraints are checked only after a solution is found. In the case that the solution violates any constraint, the constraint is added in the next iteration. Since optimization is much more computationally burdensome compared to post-optimization processing, this approach significantly reduces the computational burden of the problem, compared to solving a single iteration that includes all the constraints.

B. Formulation

1) Power Flow: As previously stated, when the net injection value is known at all buses, it is possible to calculate the line flow by using the **PTDF** matrix. This is formulated as follows:

$$\boldsymbol{F} = \boldsymbol{PTDF} \times \boldsymbol{P} \qquad (1)$$

where $\boldsymbol{P}$ is the net nodal power injection vector, which equals nodal generation minus nodal load, excluding the slack bus. In a simple case with no load shedding or over-generation, net nodal power injection can be calculated as:

$$P_{(n)} = PG_{(n)} - P^{d}{}_{(n)} \qquad \forall\ n \neq \text{Slack bus} \qquad (2)$$

and **PTDF** which is an $nl \times (nb\text{-}1)$ matrix that is calculated as:

$$\boldsymbol{PTDF} = \boldsymbol{B}_{br}\boldsymbol{A}\boldsymbol{B}^{-1} \qquad (3)$$

where $\boldsymbol{B}_{br}$ is a $nl \times nl$ matrix and calculated as:

$$B_{br(k,k')} = \begin{cases} b_{(k)}, & k = k' \\ 0, & k \neq k' \end{cases} \qquad \forall\ k, k' \in \text{L} \qquad (4)$$

In $\boldsymbol{B}_{br}$, the diagonal elements are lines susceptances, and all non-diagonal elements are zero. $\boldsymbol{A}$ is an $nl \times (nb\text{-}1)$ adjacency matrix, and its elements are determined as:

$$A_{(k,n)} = \begin{cases} +1 & \text{if line } m \text{ starts from node } n \\ -1 & \text{if line } m \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$
$$\forall\ k, \text{and } \forall\ n \neq \text{Slack bus}$$

B is an $(nb\text{-}1) \times (nb\text{-}1)$ admittance matrix (slack bus is excluded), and its elements are calculated as:

$$B_{(n,n')} = \begin{cases} -b_{(n,n')} & n \neq n' \\ \sum_{n'=1}^{nb} b_{(n,n')} & n = n' \end{cases} \qquad (6)$$

It is worth mentioning that in the same way that (1) calculates the power flow for all lines, power flow on line k can be calculated as:

$$F_{(k)} = \boldsymbol{PTDF}_{(k)} \times \boldsymbol{P} \qquad \forall\ k \qquad (7)$$

2) Line Outage: In line outage calculations, **LODF** is the main required information. **LODF** is an $nl \times nl$ matrix that can be calculated by using **PTDF**. Each element, $LODF_{(m,m')}$, determines what fraction of pre-outage power that was flowing on line m will be transferred to line m' if line m goes offline. Assuming that the "from" and "to" nodes of line m are indicated by "*fm*" and "*to*", $LODF_{(m,m')}$ is calculated as:

$$LODF_{(m,m')} = \frac{PTDF_{(m',fm)} - PTDF_{(m',to)}}{1 - (PTDF_{(m,fm)} - PTDF_{(m,to)})} \quad \forall\, k, k' \text{ and } k \neq k' \tag{8}$$

The value of (8) cannot be greater than one or less than minus one. (1) - (8) are needed in the first segment of the flowchart in Fig. 1.

3) Objective Function: The objective function for most cases can be defined as minimization of the cost function. However, in applications such as preventive operation, the goal is to minimize the lost load during a predictable hazard, putting reliability above the cost. Hence, the objective function can be defined as minimization of load shedding plus over-generation. One way to balance economic efficiency and reliability is to add the value of lost load to the generation dispatch cost, as shown in:

$$Minimize \quad \sum_s \{\pi_{(s)} \sum_t [\sum_g (c_{(g)} PG_{(s,g,t)} + c_{(g)}^{NL} u_{(s,g,t)} + c_{(g)}^{SU} v_{(s,g,t)} + c_{(g)}^{SD} x_{(s,g,t)}) + \sum_n \left(c_{(n)}^{lsh} P_{(s,n,t)}^{lsh}\right) + \sum_g (c_{(g)}^{og} P_{(s,g,t)}^{og})]\}. \tag{9}$$

In (9), the first term represents generation cost; the second term represents load shedding cost; and the third term represents over generation cost. For simplicity, this is shown in (10).

$$Minimize \quad \sum_s \{\pi_{(s)} \sum_t [\sum_g (Generation\,cost) + \sum_n (Load\ shedding\ cost) + \sum_g (Over\ Generation\,cost]\} \tag{10}$$

The generation cost itself includes power generation cost, no load cost, start-up cost, and shut-down cost. The load shedding is modeled as an expensive generator that is installed at all buses that have load. The over generation is defined as a load with expensive penalty cost installed at each generating node. Note that by removing the first term in (9) and ignoring penalty cost indices, the objective function will be the minimization of load shedding and over-generation only.

4) Constraints: The constraints of the problem are the following:

$$PG_{(g)}^{min} u_{(s,g,t)} \leq PG_{(s,g,t)} \leq PG_{(g)}^{max} u_{(s,g,t)} \quad \forall\, s, g, t \tag{11}$$

$$\sum_{h=t-UT_g-1}^{t} v_{(s,g,h)} \leq u_{(s,g,t)} \quad \forall s, g, t \tag{12}$$

$$\sum_{h=t-DT_g-1}^{t} x_{(s,g,h)} \leq 1 - u_{(s,g,t)} \quad \forall\, s, g, t \tag{13}$$

$$v_{(s,g,t)} - x_{(s,g,t)} = u_{(s,g,t)} - u_{(s,g,t-1)} \quad \forall\, s, g, t \tag{14}$$

$$P_{(s,n,t)} = \left[PG_{(s,n,t)} + P_{(s,n,t)}^{lsh}\right] - \left[P_{(s,n,t)}^{d} + P_{(s,n,t)}^{og}\right] \quad \forall\, s, n, t \tag{15}$$

$$-F_{(m)}^{max} \leq F_{(s,m,t)} \leq F_{(m)}^{max} \quad \forall\, s, t \quad \forall\, m \in \mathrm{M}_{(s)} \tag{16}$$

$$F_{(s,m,t)} = (\boldsymbol{PTDF}_{(m)} \times \boldsymbol{P}_{(s,t)}) + \sum_{o \in O_{(s,t)}} \left(PTDF_{(m,frm_{(o)})} - (PTDF_{(m,to_{(o)})}\right) FC_{(s,t,o)} \quad \forall\, s,t \quad \forall\, m \in \mathrm{M}_{(s)} \tag{17}$$

$$(\boldsymbol{PTDF}_{(o)} \times \boldsymbol{P}_{(s,t)}) - FC_{(s,t,o)} + \sum_{o' \in O_{(s,t)}} \left(PTDF_{\left(o,frm_{(o')}\right)} - (PTDF_{\left(o,to_{(o')}\right)}\right) FC_{(s,t,o')} = 0 \quad \forall\, s, t \quad \forall\, o \in \mathrm{O}_{(s,t)} \tag{18}$$

$$\sum_n [(PG_{(s,n,t)} + P_{(s,n,t)}^{lsh}) - P_{(s,n,t)}^{d} + P_{(s,n,t)}^{og}] = 0 \quad \forall\, s,\, t \tag{19}$$

$$Rmp_{(g)}^{HR} u_{(s,g,t-1)} + PG_{(g)}^{max} v_{(s,g,t)} \geq PG_{(s,g,t)} - PG_{(s,g,t-1)} \quad \forall\, s, g, t \tag{20}$$

$$Rmp_{(g)}^{HR} u_{(s,g,t)} + PG_{(g)}^{max} x_{(s,g,t)} \geq PG_{(s,g,t-1)} - PG_{(s,g,t)} \quad \forall\, s, g, t \tag{21}$$

$$u_{(s,g,t)} = u_{(s',g,t)} \quad \forall\, s, s' \in \mathrm{S} \tag{22}$$

Generation maximum and minimum limits are represented in (11). Minimum up and down times are given in (12)-(14). (15) represents the nodal net injected power calculation. It should be mentioned that (15) is the comprehensive form of (2), when load shedding and over generation are allowed. Load shedding and over-generation are modeled as injections and withdrawals, respectively. (16) ensures that the flow on the lines that should be monitored (those that violated their thermal capacity in the previous iteration) stay within the limits; (17) and (18) calculate the line power flow for such lines. (17) and (18) account for the changes in the topology of the network. A detailed explanation is given later. The power balance is defined in (19), allowing load shedding and over-generation. (20) and (21) represent the ramp-up and ramp-down limitations over the generation units. The last equation enforces the commitment status of generation units be the same for all scenarios. Equations (9) to (22) are used in the second segment of the flowchart, shown in Fig. 1

5) Multiple line outage modeling: As mentioned earlier, a single line outage can be modeled using LODF sensitivities. However, with multiple line outages, LODFs cannot be directly used as LODF depends on the network topology. After just one outage, the topology changes and LODF matrix needs to be recalculated. To overcome this challenge, flow canceling transactions are introduced in [34]–[36] for network topology optimization problem. Here, we employ the same concept to model multiple line outages. Flow canceling transactions are a pair of injections that would represent the outage of a line. The transactions are calculated such that their impact on the rest of the network resembles the outage of the line. Fig. 2 shows a meshed network with two lines that are out: O_1 and O_2. The outage of these lines are represented by a pair of flow canceling transactions: FC_1 and FC_2. Additionally, Fig. 2 shows another line in the meshed network, line m, whose flow is affected by outage of other lines. To clearly explain how flow canceling transactions are calculated, we assume that the transactions are placed on two fictional buses that are infinitely close to the "from" and "to" buses of the lines experiencing an outage. The connection between the real buses and the fictional buses are shown with dotted lines. If the flow on these dotted connections are zero, the line will be effectively out, from the viewpoint of the rest of the network. This flow will include the original line flow as well as the impact of the flow canceling transactions. The original flow on the line can be calculated using the PTDF matrix and the nodal injections: $F_O = \boldsymbol{PTDF}_{(O)} \times \boldsymbol{P}$. The impact of flow canceling transactions on the line flow can also be calculated using the PTDF matrix: $\left(1-\left(PTDF_{O,frm_{(O)}} - PTDF_{O,to_{(O)}}\right)\right)FC_O$. Note that $\left(PTDF_{O,frm_{(O)}} - PTDF_{O,to_{(O)}}\right)$ portion of the transaction will flow on the line itself and $\left(1-\left(PTDF_{O,frm_{(O)}} - PTDF_{O,to_{(O)}}\right)\right)$ of it will flow through the meshed network. This portion of the flow has to pass the dotted connections. Thus, to model a line outage, FC_O should be calculated in a way that the total flow on the dotted portion of the line becomes zero: $\boldsymbol{PTDF}_{(O)} \times \boldsymbol{P} - \left(1-\left(PTDF_{O,frm_{(O)}} - PTDF_{O,to_{(O)}}\right)\right)FC_O = 0$.

To extend this equation to the case of multiple line outages, the impact of flow canceling transactions on all the open lines should be included in the calculation of the flow canceling transactions. This will lead to a system of $|O|$ linear equations and $|O|$ unknowns, as shown in (18). It should be noted that flow canceling transactions are valid as long as the outages do not isolate a meshed part of the network. To ensure the validity of the model, the outage possibilities and their impacts on the meshed network topology are checked in Segment 1.

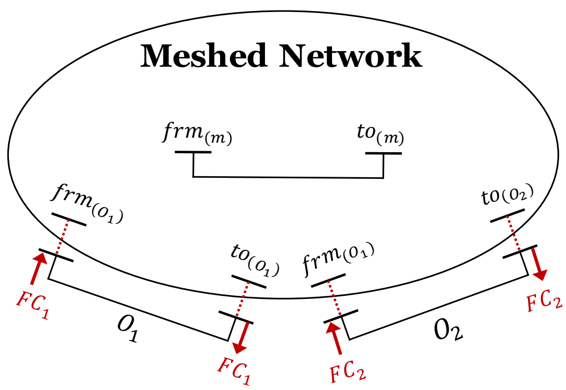

Fig. 2. Flow canceling transactions to represent multiple line outages

The advantage of (18) is that it represents the impacts of multiple line outages with a system of linear equations. Adding these equations to the optimization problem as constraints will model simultaneous outages, while keeping the complexity of the problem to a linear program. This way, the flow canceling transactions are calculated within the optimization problem, which makes the calculation of flow on other lines, e.g., line m, rather simple. For any other line in the network, the flow canceling transactions are treated simply as nodal injections, as shown in (17).

6) Post-optimization calculations in Segment 3: In segment three of the flowchart, the same equations as segment two are used with one change. In section three, all the network variables are calculated, not just the previously selected ones. For example, in power flow calculation, equations (16) and (17) are changed to (23) and (24), respectively.

$$-F_{(k)}^{max} \le F_{(s,k,t)} \le F_{(k)}^{max} \qquad \forall\, s,\, t \tag{23}$$

$$\begin{aligned} F_{(s,k,t)} = {} & (\boldsymbol{PTDF}_{(k)} \times \boldsymbol{P}_{(s,t)}) \\ & + \sum\nolimits_{out_{(s,t)}} \Big(PTDF_{(k,frm_{(out)})} \\ & - (PTDF_{(k,to_{(out)})}\Big) FC_{(s,t,out)} \end{aligned} \qquad \begin{matrix} \forall\, k \\ \forall\, s,t \\ \forall\, k \end{matrix} \tag{24}$$

This change guarantees that network violations are detected and necessary changes are implemented before algorithm termination. If a violation is found, the constraint controller matrices, such as *M,* will be updated. This update will change the optimization model, by adding additional constraints, in the next iteration and eliminate the detected violations. If no violation is found, the calculated values will be saved as the final solution to the SUC problem.

VI. Test Case

To evaluate the performance and validate the accuracy of the proposed model, it is first compared with two standard unit-commitment methods on a large network. The reason that we do not compare them in solving the SUC is that we were not able to solve any large network with multiple scenarios using other methods, with the hardware that was available to us and within an acceptable time. The selected test case network is ACTIVSg2000 (Synthetic grid on a footprint of Texas) with 2,000 buses, 540 generation units, and 3,206 transmission lines [37]. As a source of uncertainty, we used the data regarding the effect of a hurricane on the transmission lines. As the hurricane hits different parts of the network at different times, each element of the network there has a failure possibility, which is a function of time. The test system alongside the path for the hurricane is illustrated in Fig. 3.

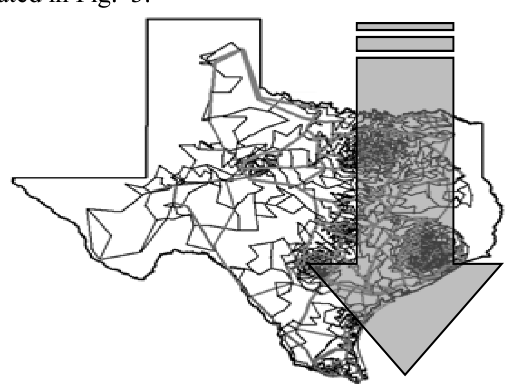

Fig. 3. Network elements and hurricane path used as a test case

A. Simulation Environment

While there are some appropriate software environment options, we chose to use Java in combination with IBM CPLEX as our simulation environment. Java is fast and has flexible memory management [38], [39]. The machine we used to run the methods utilizes Intel® Core™ i7-7700 CPU @3.60GHz as a central and only processor combined with 16.0 GB of RAM, which is configured as dual channel bandwidth @ 2.40 GHz. The software package includes Eclipse Jee ver. 4.1 and IBM CPLEX ver. 12.8-64 bit running on Windows 10 Pro.

Table 1 presents the results obtained for the standard UC problem when over-generation and load shedding is not allowed, and there is no outage during the period of study. In this table, B-θ represents the standard B-θ UC formulation; PTDF represents the fully constrained PTDF UC with all the network constraints included.

As can be seen in Table 1, the proposed algorithm has the same accuracy as other methods while it needs much less memory and time. Here, a slight difference in objective value is due to the MIP gap in CPLEX, which is set to 0.05.

TABLE 1. BENCHMARK RESULT FOR THE PROPOSED METHOD

Method	B-θ	PTDF	Proposed
Cost Function ($)	2.0186E+7	2.0186E+7	2.0185E+7
Solution Time (minute)	138	18	2
Required Random Access Memory (GigaBytes)	4	1	0.2

The main reason for the reduced calculation time is the handling of transmission constraints. The number of line constraints is decreased by 99.06% in the proposed method, as there are just 30 lines that violate the thermal capacity, not all 3,206 lines.

Adding load shedding and over-generation to the model significantly increases the number of variables and constraints for the generation units, which itself increases the solution time. It should be mentioned that the calculation time is affected by the penalty cost considered for load shedding and over-generation. For a penalty cost of 1,000 times more than the most expensive generated unit of power (about $30,000 MWhr^{-1}), the solution time increases to 4 minutes. The results for the same case as shown in Table 1 as well as load shedding and over-generation are shown in Table 2. As can be seen, the objective value is the same while the calculation time doubled.

TABLE 2. COMPARISON BETWEEN STANDARD UC AND WHEN LOAD SHEDDING IS ALLOWED AT A HIGH COST

	Objective Value ($)	Unserved Load (MW)	Solution Time (Sec)	Iterations Required
Standard UC	2.0185E+7	0	120	3
UC + Load shedding	2.0185E+7	0	250	4

Adding scenarios to the stochastic UC is evaluated in the next step. In order to test the stochastic UC performance, we considered ten scenarios. Ten scenarios describe ten possible conditions regarding the future uncertainties in the network.

We used failure prediction data to generate scenarios in a way that the first scenario represents the best possible case (no outage), and the last scenario represents the worst possible case in term of outages. The total number of affected lines by the hurricane is around 100.

Having this test case, we aim to investigate two research questions: 1) is the algorithm computationally efficient? and 2) does it significantly improve reliability?

In term of performance in the calculation, the solution time is about four hours, and it consumes less than Five GB of RAM, while there is more free available memory on the system. An important note here is that as the algorithm uses an iterative method to detect bounded variables and put them in the constraints, the solution time and memory is highly dependent on outages (number of outages and their locations) and scenarios. We tried different cases from a few outages to many outages with different scenario creation methods. While the solution time was different for each case, it was never more than ten hours for ten scenarios.

To answer the second question, we ran a Monte Carlo simulation using the failure possibility data we had from a hurricane, to check how effective the developed algorithm is. Results are shown in Table 3.

TABLE 3. COMPARISON BETWEEN BUSINESS AS USUAL AND USING PREVENTIVE SUC FOR DAY-AHEAD GENERATION PLAN

	Expected Unserved Load (MWhr)	Expected Cost ($)
Business as Usual	39,393	1.190E+9
Preventive SUC	17,188	0.531E+9

A significant finding here is that preventive SUC can reduce the expected unserved load by 56.4%. In cases such as hurricanes, reducing power outages by a factor of 50% is rather significant. Note that such reduction in outages was achieved using an enhanced implementation of SUC, which is tractable in a large-scale Texas test case.

CONCLUSION

While there are many methods to solve the stochastic unit commitment problem, none of them is suitable for large networks and multiple uncertain line outage possibilities. In this paper, for the first time, we developed a tractable method to solve SUC when there is a possibility for the failure of a large number of transmission lines. The method uses shift factors for rapid calculation of power flows and flow canceling transactions for efficient modeling of multiple line outages. Additionally, to keep the size of the problem manageable, critical constraints are detected and added iteratively. As one application for the proposed algorithm, a preventive day-ahead unit commitment model was developed for Texas 2000-bus test system, affected by a hypothetical hurricane. The simulation results showed that it is possible to reduce the power outage and increase system reliability significantly by implementing the proposed method. The results also showed that the proposed method is tractable on large-scale systems.

VII. FUTURE WORK

To effectively use and solve preventive SUC, there are two main concerns that need to be addressed. First, a tractable method and formulation are required to handle large-scale real-world systems . Second, an efficient scenario selection method is needed to select a small but representative subset out of all the possible scenarios. In applications such as preventive SUC, to minimize the load loss during a hurricane, the number of all possible scenarios can easily be larger than the number of atoms in the earth. Obviously, all of the scenarios cannot be modeled. The first concern was addressed in this paper and the second one requires further research in the future.

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